# Arithmetic in the additive power monoid of natural numbers

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Let $\mathbb{N} = (\mathbf{N}, +)$ be the additive monoid of natural numbers. The set of all non-empty finite subsets of $\mathbf{N}$ is then made into a non-cancellative, atomic, commutative monoid, which we denote by $\mathcal{P}_{\text{fin}}(\mathbb{N})$ and plays a central role in combinatorial number theory [**?**, **?**], by the binary operation of *set addition*:

$$(X, Y) \mapsto \{x + y : (x, y) \in X \times Y\}.$$

We will discuss some aspects of the arithmetic of $\mathcal{P}_{\text{fin}}(\mathbb{N})$, with an emphasis on several invariants (sets of lengths, distances, elasticities, etc.) that are of primary importance in factorization theory [**?**].

## Riferimenti bibliografici

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