

# ON PHASE-SEPARATION PHENOMENA IN ELLIPTIC SYSTEMS WITH STRONG COMPETITION

NICOLA SOAVE

ABSTRACT. The study of the asymptotic behaviour of singularly perturbed equations and systems of elliptic type is a very broad and active subject of research. In recent years, a lot of interest has been given to systems of equations of competing densities, coming from chemical, biological, physical or purely mathematical applications. Typical examples of such systems fit under the comprehensive model

$$-\Delta u_i = f_i(x, u_i) - \beta g_i(u_1, \dots, u_k) \quad \text{in } \Omega \subset \mathbf{R}^N,$$

where the functions  $g_i$ , modelling the interaction between the densities, can assume different shapes according to the underlying phenomena.

Great efforts have been directed to the description of the asymptotic behavior of the solutions when the competition parameter  $\beta$  diverges; the main issues which have been investigated are:

- (1) to develop a common regularity theory for the solutions of the system, which is independent of the strength of the competition  $\beta > 0$ ;
- (2) to investigate under which assumptions one can guarantee convergence of the solutions to some limit profile;
- (3) to study the regularity of the class of limit profiles, both in terms of the densities and in terms of the emerging free boundary problem;
- (4) to give qualitative properties and precise estimates of such convergence.

In this introductory talk we briefly discuss the main available results and the many open questions which still have to be solved.

POST-DOC, JUSTUS-LIEBIG-UNIVERSITÄT, GIESSEN, GERMANY;

*E-mail address:* nicola.soave@gmail.com