SYSTEMS OF FILTERS

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ABSTRACT. The most powerful large cardinal axioms known postulate the existence of an elementary map (a map preserving the truth of first-order formulas) from the universe V into a subclass M (with certain desired properties). These axioms are able to solve several undecidable statements coming from different areas of mathematics, thus settling as fundamental tools in the quest for new axioms limiting the incompleteness phenomena. However, the existence of an elementary map is a second-order statement, thus putting these axioms outside of the first-order setting where the completeness theorem holds.

To overcome this issue, several set-sized object have been introduced (ideals, extenders, towers) which are able to induce a class-sized elementary map with the desired properties. We follow this attempt introducing $\mathcal{C}$-*systems of filters*, a new notion which is able to simultaneously generalize both extenders and towers of normal ideals (both in the classical or generic setting). This notion allows a more compact treatment of the main theory of extenders and towers, and preserves all the main properties of both of them.

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