GENERALISED NUMBER SYSTEMS

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ABSTRACT. Let Λ be a lattice of $\mathbb{R}^k$, $\mathcal{D} \subseteq \Lambda$ a finite subset containing 0, $M : \Lambda \to \Lambda$ an invertible linear operator of $\mathbb{R}^k$ mapping Λ to itself. Then $(\Lambda, M, \mathcal{D})$ is called a *(generalised) number system* if every element $z \in \Lambda$ has a unique finite representation of the form

$$z = d_0 + d_1 M + \dots d_l M^l = (d_0, d_1, \dots, d_l)_M$$

with $d_0, \dots, d_l \in \mathcal{D}$; M is the *base* or *radix*, $\mathcal{D}$ is the *digit set*.

In this talk we shall survey the main computational problems concerning generalised number systems.

- Decision: given $(\Lambda, M, \mathcal{D})$, is it a number system? Is there an effective algorithm? Is the problem NP-hard?
- Classification: if $(\Lambda, M, \mathcal{D})$ is not a number system, how can we identify all the periodic elements (i.e. those elements that do not admit a finite expansion)?
- Construction: given Λ and M, is there any digit set $\mathcal{D} \subseteq \Lambda$ s.t. $(\Lambda, M, \mathcal{D})$ is a number system? If yes, how many digit sets are there? How to construct them?
- Length: given a number system $(\Lambda, M, \mathcal{D})$, what is the length of the expansion of $\eta \in \Lambda$?

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