FROM LIE GROUPS TO LIE GROUPOIDS

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ABSTRACT. A complete vector field $X \in \mathfrak{X}(M)$ defines a global flow $\phi : \mathbb{R} \times M \to M$, or, equivalently, a Lie group action of $(\mathbb{R}, +)$ on M; however, when X is not complete, the flow is not globally defined and the group law $\phi_s \circ \phi_t = \phi_{s+t}$ is not always true.

If we move from this algebraic point of view to a categorical one, we can see that the first case corresponds trivially to a category with objects M and arrows $\mathbb{R} \times M$ (where (t, x) goes from x to $\phi_t(x)$), characterised by the property that every arrow (t, x) has an inverse $(-t, \phi_t(x))$. In the second case we cannot define a group structure on $\{\phi_t\}_t$, but we obtain nevertheless a category with invertible arrows: this will be our guiding example for the concept of Lie groupoid.

After formalising the definition, we will provide an overview of several Lie groupoids, which are naturally related with well-known objects of differential geometry (such as group actions, principal bundles, connections and metrics) and help to describe them better. We will conclude the parallelism with Lie theory introducing Lie algebroids (which will be vector bundles instead of vector spaces) and showing how Lie groupoids "integrate" them.

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