Welcome Home Workshop 2014

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TITOLO: Borel Conjecture and dual Borel Conjecture

Abstract

I will talk about a consistency result in set theory. We will see that it is consistent that the Borel Conjecture and the dual Borel Conjecture hold simultaneously. This is a result by Goldstern, Kellner, Shelah and Wohonfsky (2013). The Borel Conjecture (BC) says: *every strong measure zero set is countable.* The dual Borel conjecture (dBC) says: *every strongly meager set is countable.* A set of reals $X \subseteq 2^\omega$ is *strong measure zero* if for all $f : \omega \to \omega$ there are intervals I_n of measure $\leq 1/f(n)$ covering X. A set X is *strongly meager* if every set of Lebesgue measure 1 contains a translate of X.

Further explanation

Using the method of forcing, Laver showed in 1976 that BC is consistent. Carlson proved the consistency of dBC in 1983. While the first attempts to force BC+dBC combining Laver's and Carlson's constructions faced strong obstacles, it is possible [1] to mix these two "generically" and prove that *if ZFC is consistent, then ZFC+BC+dBC is consistent.*

ZFC (Zermelo-Fraekel axioms and the axiom of choice) is a reasonable set of axioms for mathematics from which most mathematical theorems can be driven. However, due to Goedel's incompleteness theorem (1931) there are statements that cannot be deduced from ZFC. That is, a sentence ϕ can be *independent* from ZFC; i.e. neither ϕ nor $\neg\phi$ follows from ZFC. To show that ϕ is unprovable from ZFC, we need a model of ZFC, in which ϕ is false. Recall that a set of axioms is *consistent* if and only if it has a model. Invented in 1963 by Paul Cohen, *forcing* is a powerful technique for obtaining models of ZFC with desired properties: the idea is to start with a given model M of ZFC and extend it by adjoining an object G, the generic filter that is a subset of some partially ordered set P. The resulting model $M[G]$, the minimal model containing M and G, also satisfies ZFC and G can be constructed so that ϕ is false in $M[G]$.

References

[1] Saharon Shelah Wolfgang Wohofsky Martin Goldstern, Jakob Kellner. *Borel Conjecture and Dual Borel Conjecture*. 2013.