A new definition of firing time in stochastic (leaky) integrate–and–fire neuron models

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The topic

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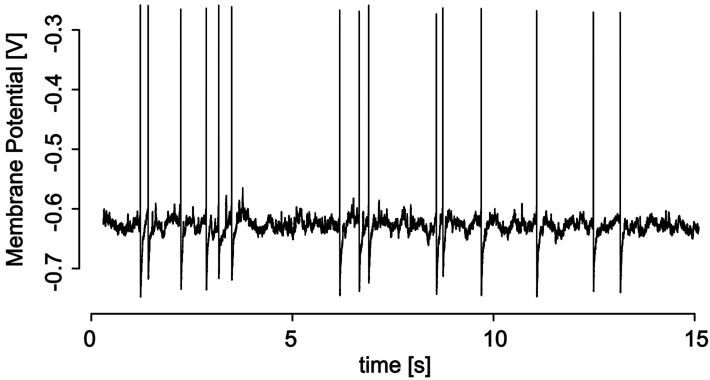
- The goal is to introduce a new definition of **firing time**

The membrane potential



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- Stochastic IF neuronal models are characterised by the following features:
  - the **stochastic process** that describes the membrane potential between two consecutive spikes
  - the **firing mechanism** that produces the action potential (not included in the model)
  - the **resetting** condition

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$$dV_t = a(t, V_t)\ dt + b(t, V_t) dW_t, \quad V_0 = v_0$$

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- **Remarks:**
  - the threshold S is a **firing** threshold
  - no spike, no crossing of the threshold
  - the threshold level is the maximum depolarisation achieved between two spikes

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- **Remark:**
  - after resetting the process starts from scratch
  - if we reset both **time** and **space** the sequence of **interspike intervals** is a random sample drawn from T
  - ... and the **spike train** is a renewal point process

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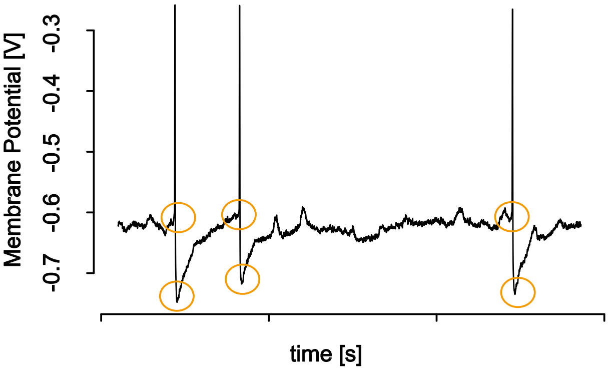

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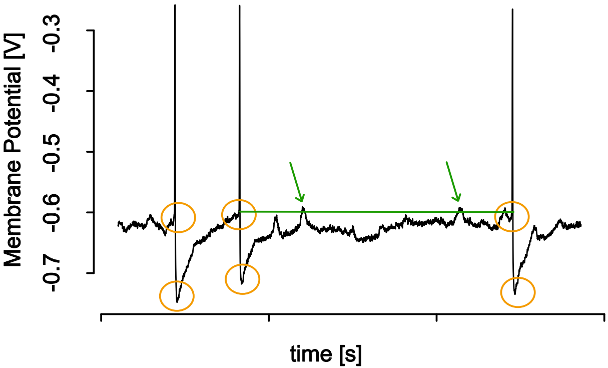

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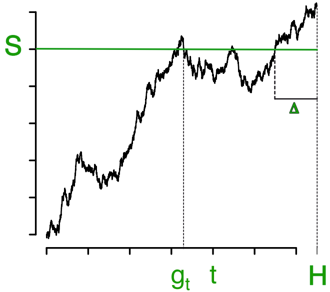
- The model is misspecified!
  - the membrane potential may cross the threshold several times before firing
  - the threshold cannot be considered constant

The new firing time

The new firing time

- Let the cell fire as its membrane potential reaches the threshold level S **and constantly remains above this level** for a time interval equal to a fixed constant Δ



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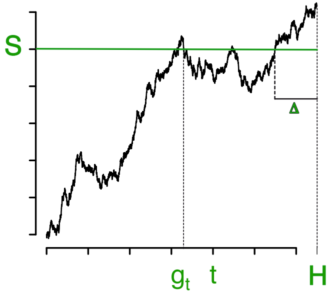

$$H = \inf\{t : \mathbf{1}_{V_t \geq S} \cdot (t - g_t) \geq \Delta\}$$

The new firing time: the perfect integrator

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- In the case of the **perfect integrator**, i.e. the membrane potential process is a Wiener process

$$dV_t = \mu dt + \sigma dW_t, \quad V_0 = 0$$

we can derive the Laplace transform

$$\mathbb{E}\left(\mathrm{e}^{-\lambda H}\right)$$

The new firing time: the leaky integrator

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- In the case of the **leaky integrator**, i.e. the membrane potential process is an Ornstein Uhlenbeck process

$$dV_t = \left(-\frac{V_t}{\tau} + \mu\right) dt + \sigma dW_t, \quad V_0 = 0$$

we are still working in the framework of the general **excursion theory**.

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:)