The transfer function of the LIF model: A reduction from colored to white noise

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The assessment of the stability of neuronal networks, the emergence of oscillations and correlated activity rely on the response properties of a neuron to a modulation of its input, i.e. the transfer function. For the leaky integrate-and-fire neuron model exposed to white noise the transfer function has been derived analytically [1, 2]. The decay time of a few milliseconds for a postsynaptic current amounts to the synaptic noise not being white, but rather having higher power at low frequencies. The effect of such colored noise on the response properties has been studied intensively at the beginning of the last decade [3, 4],. Analytical results were derived only in the high frequency limit while it was claimed that up to moderate frequencies the first order correction in the time constant of the noise vanishes. Complementary we here show that the first order correction exists and derive a novel analytical expression valid up to moderate frequencies.

The analysis of such synaptic filtering comes along with considerable difficulties, since it adds a dimension to the governing Fokker-Planck equation. We overcome these complications by developing a general method of reduction to a lower dimensional effective system, respecting the details of the noise in the boundary conditions [5]. For the stationary case this reduction was derived earlier by a perturbative treatment of the arising boundary layer problem [5]. Here we extend this study to the dynamic case and obtain a fundamental new insight: Colored-noise approximations for stationary but more importantly also for dynamic quantities are directly obtained by shifting the location of the boundary conditions in the white-noise solutions. We compare the analytical results to direct simulations and investigate the effect of colored noise on the transfer properties at moderate frequencies.

The novel method [6] can be applied to generic first order stochastic differential equations that are driven by fast colored noise. The method amounts to a reduction to a one dimensional system, rendering many problems tractable that have to date been considered beyond the realm of analytical solutions.

References

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