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Automated parameters for troubled-cell indicators using outlier detection

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In general, solutions of nonlinear hyperbolic PDE's contain shocks or develop discontinuities. One option for improving the numerical treatment of the spurious oscillations that occur near these artifacts is through the application of a limiter. The cells where such treatment is necessary are referred to as troubled cells, which are selected using a troubled-cell indicator. Examples are the KXRCF shock detector, the minmod-based TVB indicator, and the modified multiwavelet troubled-cell indicator.
The current troubled-cell indicators perform well as long as a suitable, problem-dependent parameter is chosen. An inappropriate choice of the parameter will result in detection of too few or too many elements. Detection of too few elements leads to spurious oscillations, since not enough elements are limited. If too many elements are detected, then the limiter is applied too often, and therefore the method is more costly and the approximation smooths out after a long time. The optimal parameter is chosen such that the minimal number of troubled cells is detected and the resulting approximation is free of spurious oscillations. In general, many tests are required to obtain this optimal parameter for each problem.
In this presentation, we will see that the sudden increase or decrease of the indicator value with respect to the neighboring values is important for detection. Indication basically reduces to detecting the outliers of a vector (one dimension) or matrix (two dimensions). This is done using Tukey's boxplot approach to detect which coefficients in a vector are straying far beyond others [2].
We provide an algorithm that can be applied to various troubled-cell indication variables. Using this technique, the problem-dependent parameter that the original indicator requires, is no longer necessary, as the parameter will be chosen automatically.
We will apply this technique to the modified multiwavelet troubled-cell indicator [3, 4], which can be used to detect discontinuities in (the derivatives of) the DG approximation. Here, Alpert's multiwavelet basis is used [1]. We will use either the original indicator (with an optimal parameter), or the outlier-detection technique. In that way, the performance of the new technique can be easily compared to the current method.

References

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