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# A comparison between well-balanced numerical approaches for the simulation of shallow flows on bottom discontinuities

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In last two decades, in the context of the Shallow Water Equations (SWE) numerical integration, the efforts of many researchers were devoted to conceive techniques for the exact preservation of motionless state over non-flat bottom. Recently, such efforts are mainly oriented to the proper treatment of the bottom discontinuities and to the exact preservation of moving water steady flow. The well-known definition of well-balanced scheme for quiescent flow is therefore extended to steady flows. At the present state of art a large number of techniques for the well-balancing of a SWE model in the case of a quiescent flow exists whilst very few approaches for the well-balancing of a moving steady flow are available.
In this work we give a contribution to the well-balancing of SWE models for moving steady flow, indicating how some key elements of the standard approaches have to be changed to improve the overall behavior of the schemes. We have focused our attention on both the classical finite volume formulation and the path-conservative formulation, inside a unified context represented by third-order accurate discontinuous Galerkin schemes. In particular, a comparison between five numerical treatments of the bottom discontinuities is presented.
We consider three widespread approaches that perform well if the motionless state has to be preserved. First, a simple technique, which consists in a proper initialization of the bed elevation that imposes the continuity of the bottom profile is taken into account [1]. Then, we consider the hydrostatic reconstruction method [2] and a path-conservative scheme based on a linear integration path [3].
We than consider two further approaches (the former characterized by a limited diffusion and the latter original) which are promising for the preservation of a moving-water steady state. The former is obtained modifying the hydrostatic reconstruction as suggested in [4]. This method is characterized by a correction of the numerical flux based on the local conservation of the total head. The last model is obtained improving the path-conservative scheme using a curvilinear path. The non-linear path is defined imposing the local conservation of the total head and the discharge at the cell-interfaces.
Several test cases are used to verify the accuracy, the well-balancing, the behavior in simulating a quiescent flow and the resolution of the models in simulating unsteady flows. A specific test case is also introduced to highlight the difference between the five schemes when a steady moving flow interacts with a bottom step.

## References

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