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On the Relation between WENO3 and Third-Order Limiter Functions in Finite Volume Methods

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We are interested in the numerical solution of hyperbolic conservation laws on the most local compact stencil consisting of only nearest neighbors. In the Finite Volume setting, the main challenge is the reconstruction of the interface values which are crucial for the definition of the numerical flux functions and thus, for the order of accuracy of the resulting scheme.

Often, the functions of interest contain smooth parts as well as large gradients, discontinuities, or shocks. Treating such functions with high-order schemes may lead to undesired effects such as oscillations. However, what is required is a solution with sharp discontinuities while maintaining high-order accuracy in smooth regions. One possible way of achieving this is the use of limiter functions in the MUSCL framework which switch the reconstruction to lower order when necessary. Another possibility is the third-order variant of the WENO family, called WENO3 which was introduced by Jiang and Shu [3].

In this work, we will recast both methods in the same framework to demonstrate the relation between Finite Volume limiter functions and the way WENO3 performs limiting. Special attention is given to the limiter function developed by Čada and Torrilhon [2], which is based on the local double logarithmic reconstruction function of Artebrant and Schroll [1]. The limiter contains a decision criterion which distinguishes between discontinuities and smooth extrema, containing a parameter r, which is the radius of a so-called asymptotic region, [2]. Unfortunately, r remains unspecified. Our analysis shows that r can be coupled to the solution. Thus, our newly-developed limiter function does not require an artificial parameter. Instead, it uses only information of the initial condition.

We compare our insights with the formulation of the weight-functions in WENO3. The weights as defined in [3] contain a parameter ε which was originally introduced to avoid the division by zero. However, we will show that ε has a significant influence on the behavior of the reconstruction and relating ε to r allows us to give a clarifying interpretation.

The comparison of both approaches proves fruitful for either one and allows further insight into limiter functions.

[1] Artebrant, R. and Schroll, H.J. Conservative logarithmic reconstructions and finite volume methods. *SIAM Journal on Scientific Computing*, **27(1)**, 294-314, 2005.

[2] Čada, M., and Torrilhon, M. Compact third-order limiter functions for finite volume methods. *Journal of Computational Physics*, **228(11)**, 4118-4145, 2009.

[3] Jiang, G. S., Shu, C. W. Efficient implementation of weighted ENO schemes. *Journal of Computational Physics*, **126(1)**, 202-228, 1996.