Miniworkshop

Kernel-based methods and function approximation

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Book of Abstracts

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Lagrange interpolation on unbounded intervals and applications

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In this talk it is proposed a new Lagrange interpolation process $\{\mathcal{L}_m(f)\}_{m\in N}$ based on the knots $\{\{\zeta_k\}_{k=1}^m\}_{m\in N}$ related to the matrix of Laguerre zeros $\{\{x_{m,k}(w_\alpha)\}_{k=1}^m\}_{m\in N}$, $\quad w_\alpha(x)=e^{-x}x^\alpha$.

As an application it will be shown how to approximate the Hilbert transform

$$\mathcal{H}(fw_\alpha,t):=\int_0^{+\infty}\frac{f(x)}{x-t}w_\alpha(x)dx, \tag{1}$$

by means of the Lagrange interpolation sequence $\{\mathcal{L}_m(f)\}_{m\in N}$ combined with the Truncated Gauss-Laguerre rule [1]. This procedure is an improvement in some sense of the method proposed in [2]. Some numerical experiments will be shown to confirm the efficiency of the numerical procedure.

References

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Multinode operators for Birkhoff interpolation

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The Hermite interpolation problem consists of determining a continuous function, belonging to a given interpolation space, such that its values and successive derivatives at each interpolation node match certain given values, called data values. The Birkhoff interpolation problem is a generalization of the Hermite one, which differs only for the fact that selected derivatives data may not be successive (such data are then called lacunary or irregular). Moreover, while an Hermite interpolation problem can be always solved, a Birkhoff interpolation problem is not always solvable even in the appropriate polynomial space. We propose a method that split up the initial problem in subproblems having a unique polynomial solution and use multinode rational basis functions in order to obtain a global interpolant. The study of the remainder term allows to give information on the order of the approximation of the resulting operator.

Multiscale Approximation by Radial Basis Functions

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Radial basis functions (RBFs) are a popular meshfree discretisation method. They are used in various areas comprising, for example, scattered data approximation, computer graphics, machine learning, aeroelasticity and the geosciences.

The approximation space is usually formed using the shifts of a fixed basis function. This simple approach makes it easy to construct approximation spaces of arbitrary smoothness and in arbitrary dimensions.

Multiscale RBFs employ radial basis functions with compact support. In contrast to classical RBFs they do not only use the shifts of a fixed basis function but also vary the support radius in an orderly fashion. If done correctly, this leads to an extremely versatile and efficient approximation method.

In this talk, I will discuss the basic ideas of multiscale RBFs, I will give and analyse an explicit algorithm for the reconstruction of multivariate functions from scattered data. After that, I will discuss how multiscale RBFs can be used for data compression, for the resolution of different scales in the target function and how they can be used to solve partial differential equations numerically.

Kernel-based Image Reconstruction from Scattered Radon Data by Anisotropic Positive Definite Functions

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In this talk we present a novel kernel-based reconstruction method for image reconstruction from scattered Radon data. Our reconstruction relies on generalized Hermite-Birkhoff interpolation by positive definite kernel functions. For radially symmetric kernels, however, a straightforward application of generalized Hermite-Birkhoff interpolation fails to work (cf. [1, 2]). For the wellposedness of the reconstruction scheme, we introduce *anisotropic* positive definite kernels, which are symmetric but not radially symmetric. We prove the well-posedness of the resulting reconstruction scheme. Moreover, we introduce a novel concept for the construction of anisotropic positive definite kernels, before we develop concrete examples for suitable combinations of radial weight functions and commonly used positive definite kernels. This leads to a very flexible image reconstruction method, which works for arbitrary distributions of Radon lines and allows also to select the most significant ones by a thinning approach based on Newton's bases. The good performance of the proposed kernel-based image reconstruction method is supported by numerical examples and comparisons.

References

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Efficient algorithms for kernel-based partition of unity interpolation with applications in geometric design

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Recently, in approximation theory, the Partition of Unity Method (PUM), performed with local Radial Basis Function (RBF) interpolants, has been proved to be an effective numerical tool for solving *large* scattered data interpolation problems, [3, 4]. In such local approach, the efficient organization of the data among the different subdomains turns out to be the crucial step.

Thus, starting from the results shown in [1], we propose a versatile software for bivariate and trivariate interpolation which makes use of a new partitioning structure, named *block-based partitioning structure*, and a novel related searching procedure. Specifically, such structure, built *ad hoc* for the PUM, enables us to run the searching procedure in constant time complexity. An extensive complexity analysis supports our findings. Moreover, in order to show the flexibility of the proposed method, we also provide applications in population dynamics, [2].

References

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Estimating the tumor growth: a RBF-PSO based method

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Prostate cancer is one of the most common tumors in elderly men. Fortunately it grows slowly and the relapse after prostatectomy could be diagnosed in an early stage by dosing the Prostate Specific Antigen (PSA). The so called Phenomenological Universalities [1, 2] can be effectively applied to model the tumor growth. Our aim is to validate such model testing it by challenging real data with popular numerical tools. Specifically, the use of low regularity Radial Basis Functions (RBFs) enables us to reconstruct the PSA curve for each patient [3]. The critical value of the shape parameter of the basis function is taken into account and selected via a cross-validation scheme. The PSA curve is later used to estimate the growth rate of the prostate cancer, via the so-called Particle Swarm Optimization (PSO) technique, enabling us to evaluate the risk level of the disease.

References

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