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A staggered semi-implicit arbitrary high order discontinuous Galerkin method for the Incompressible Navier-Stokes equations

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We present a new arbitrary high order accurate semi-implicit discontinuous Galerkin (DG) method for the solution of the incompressible Navier-Stokes equations on staggered unstructured curved meshes.
The scheme is based on the general ideas proposed in [1, 2] for the arbitrary high order in space and it is then extended to a fully space-time high order in [3] for the two dimensional case.
While the discrete pressure is defined on the primal grid, the discrete velocity field is defined on an edge-based dual grid. The method is designed in such a fashion that the entire velocity vector is defined on the edge-based dual control volumes. Formal substitution of the discrete momentum equation into the discrete continuity equation yields a sparse block four-diagonal linear equation system for the scalar pressure. The high order in time is then achieved by considering a pressure gradient formulation on space-time control volumes and introducing a Picard iteration to update the nonlinear convection.
The flexibility of high order DG methods on curved unstructured meshes allows to discretize even complex physical domains with rather coarse grids. For the space high order case the main system for the pressure results symmetric and at least semi-positive definite and hence it allows to use fast iterative linear solver such as CG method. In addition, all the volume and surface integrals needed by the scheme depend only on the geometry and the polynomial degree of the basis and test functions and can therefore be precomputed and stored in a preprocessing stage. This leads to a significant saving in terms of computational effort for the time evolution part. In this way also the extension to a fully curved isoparametric approach becomes natural and affects only the preprocessing step.
The method can then be extended to a fully three-dimensional arbitrary high order scheme based on staggered tetrahedral-hexahedral elements. Also in this case, the resulting main system for the pressure is symmetric and in general semi-positive definite up to the first order in time, extendable to the second order using a Crank-Nicolson procedure. On the contrary, we loose the symmetry but we get arbitrary high order results also in time, crucial to obtain good solutions even when high unsteady problems or huge time steps have been considered.

References

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