Miniworkshop
High order reconstruction and
well balancing techniques
for hyperbolic conservation
and balance laws
April 15–16, 2015, Torino, Italy

Smoothness-Increasing Accuracy-Conserving filtering for discontinuous Galerkin methods with source terms

Jennifer K. Ryan [a,*]

[a] University of East Anglia
Norwich NR4 7TJ, United Kingdom

*Jennifer.Ryan@uea.ac.uk

Discontinuous Galerkin (DG) methods exhibit hidden accuracy which should be used to further the betterment of the approximation. One method is to implement a convolution kernel approach that improves the order of accuracy from $k+1$ to order $2k+m$ for time-dependent linear convection-diffusion equations, where k is the highest degree polynomial used in the approximation and m depends upon the choice of the numerical flux.

In this talk, we discuss the accuracy enhancing capabilities of the Smoothness-Increasing Accuracy-Conserving (SIAC) filter for DG solutions to hyperbolic equations with smooth source terms. We discuss theoretical and computational results and the possibilities for accuracy enhancement. We implement the Smoothness-Increasing Accuracy-Conserving filter that enhances accuracy by utilizing information contained in the negative-order norm. The local postprocessing technique that makes use of the information contained in the negative-order norm was originally developed by Bramble and Schatz in the context of continuous finite element methods for elliptic problems [1]. They demonstrated that it is possible to construct a better approximation by convoluting the finite element solution with a local averaging operator in the neighborhood of a point x, where the convergence in the negative-order norms was higher than L^2-norm. Cockburn, Luskin, Shu, and Suli established a framework to apply this technique to linear hyperbolic equations in the context of the Discontinuous Galerkin methods [2]. Numerical experiments showed that the post-processing had a positive impact on nonlinear hyperbolic equations [3, 4]. This technique is labelled as a SIAC filter and was extended to nonuniform meshes, higher-order derivatives, and as a filtering technique to improve the visualization of streamlines [5, 6, 7].

References

[1] J.H. Bramble, A.H. Schatz. Higher order local accuracy by averaging in the finite element method. *Mathematics of Computation,* **31**: 94-111, 1977.
[2] B. Cockburn, M. Luskin, C.-W. Shu, E. Suli. Enhanced accuracy by postprocessing for finite element methods for hyperbolic equations. *Mathematics of Computation,* **72**: 577-606, 2003.
[3] L. Ji, Y. Xu, J.K. Ryan. Accuracy Enhancement for the linear convection-diffusion equation in multiple dimensions. *Mathematics of Computation,* **81**: 1929-1950, 2012.
[4] L. Ji, Y. Xu, J.K. Ryan. Negative-order norm estimates for nonlinear hyperbolic conservation laws. *Journal of Scientific Computing,* **54**: 269-310, 2013.
[5] J.K. Ryan, B. Cockburn. Local Derivative Post-processing for the discontinuous Galerkin method. *Journal of Computational Physics,* **228**: 8642 - 8664, 2009.
[6] J.K. Ryan, C.-W. Shu, H. Atkins. Extension of a post-processing technique for the discontinuous Galerkin method for hyperbolic equations with application to an aeroacoustic problem. *SIAM Journal on Scientific Computing,* **26**: 821-843, 2005.
[7] M. Steffan, S. Curtis, R.M. Kirby, J.K. Ryan. Investigation of Smoothness Enhancing Accuracy-Conserving Filters for Improving Streamline Integration Through Discontinuous Fields. *IEEE-TVCG,* **14**: 680 - 692, 2008.