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Accuracy analysis of finite difference Weighted Essentially Non Oscillatory schemes and boundary extrapolation techniques for complex domains

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Finite difference WENO schemes [4, 5] have become an efficient method for the approximate solution of multidimensional hyperbolic conservation laws. These schemes follow a method of lines strategy, for which the spatial discretization is obtained by numerical differentiation of reconstruction of fluxes. Higher order accuracy can then be obtained from highly accurate reconstructions.
In [1] we analyze Jiang-Shu's smoothness indicators [4], for any stencil length, and their derived weights, obtaining that they have accuracy properties that give maximal order schemes. In [2] we analyze the faster converging weights proposed in [8] to conclude that the accuracy of the schemes obtained from these weights near discontinuities deteriorate with respect to classical ENO schemes. We obtain a modification of these weights with both faster converging properties at smooth regions, regardless of neighboring extrema, and enough accuracy near discontinuities.
Finite difference WENO schemes can be quite readily designed as long as the underlying mesh is an equispaced Cartesian mesh. In this context, the application of suitable numerical boundary conditions for hyperbolic conservation laws on domains with complex geometry has become a problem with certain difficulty that has been tackled in different ways according to the nature of the numerical methods and mesh type ([6, 7]). In [3] we propose an extrapolation technique on structured Cartesian meshes (which, as opposed to non-structured meshes, can not be adapted to the morphology of the domain boundary) of the information in the interior of the computational domain to ghost cells. This technique is based on the application of Lagrange interpolation with a previous filter for the detection of discontinuities that permits a data dependent extrapolation, with higher order at smooth regions and essentially non oscillatory properties near discontinuities.

References

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