

A posteriori subcell limiting of the discontinuous Galerkin finite element method for hyperbolic conservation laws

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We propose a novel a posteriori finite volume subcell limiter technique for the Discontinuous Galerkin finite element method for nonlinear systems of hyperbolic conservation laws in multiple space dimensions that works well for arbitrary high order of accuracy in space and time and that does not destroy the natural subcell resolution properties of the DG method. High order time discretization is achieved via a one-step ADER approach that uses a local space-time discontinuous Galerkin predictor method to evolve the data locally in time within each cell.

Our new limiting strategy is based on the so-called MOOD paradigm, which a posteriori verifies the validity of a discrete candidate solution against physical and numerical detection criteria after each time step. Here, we employ a relaxed discrete maximum principle in the sense of piecewise polynomials and the positivity of the numerical solution as detection criteria. Within the DG scheme on the main grid, the discrete solution is represented by piecewise polynomials of degree N . For those troubled cells that need limiting, our new limiter approach recomputes the discrete solution by scattering the DG polynomials at the previous time step onto a set of $N_s = 2N + 1$ finite volume subcells per space dimension. A robust but accurate ADER-WENO finite volume scheme then updates the subcell averages of the conservative variables within the detected troubled cells. The recomputed subcell averages are subsequently gathered back into high order cell-centered DG polynomials on the main grid via a subgrid reconstruction operator. The choice of $N_s = 2N + 1$ subcells is optimal since it allows to match the maximum admissible time step of the finite volume scheme on the subgrid with the maximum admissible time step of the DG scheme on the main grid, minimizing at the same time also the local truncation error of the subcell finite volume scheme. It furthermore provides an excellent subcell resolution of discontinuities.

Our new approach is therefore radically different from classical DG limiters, where the limiter is using TVB or (H)WENO reconstruction based on the discrete solution of the DG scheme on the main grid at the new time level. In our case, the discrete solution is recomputed within the troubled cells from the old time level using a different and more robust numerical scheme on a subgrid level.

We illustrate the performance of the new a posteriori subcell ADER-WENO finite volume limiter approach for very high order DG methods via the simulation of numerous test cases run on Cartesian grids in two and three space dimensions, using DG schemes of up to *tenth* order of accuracy in space and time $N = 9$. The method is also able to run on massively parallel large scale supercomputing infrastructure, which is shown via one 3D test problem that uses 10 billion space-time degrees of freedom per time step.

We will furthermore show extensions of the present approach to space-time adaptive Cartesian grids (AMR).

References

- [1] M. Dumbser, O. Zanotti, R. Loubere and S. Diot. A Posteriori Subcell Limiting of the Discontinuous Galerkin Finite Element Method for Hyperbolic Conservation Laws. *Journal of Computational Physics* **278**:47-75, 2014.