Generalized spline spaces and their optimal basis

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A generalized spline space is a space of piecewise functions, locally spanned by functions of polynomial and non-polynomial type, such as trigonometric or hyperbolic. Therefore generalized splines are a superset of polynomial splines. Compared to the latter, they are superior for their capacity of reproducing fundamental functions and for their effectiveness in handling shape preserving approximation problems. At the same time, generalized splines can be quite easily differentiated and integrated, which is their main advantage over Non-Uniform Rational B-Splines (NURBS). Despite being initially meant for Geometric Modeling application, in the latest years their interest has been mainly connected to Multiresolution Analysis and Isogeometric Analysis.

A generalized spline space is suitable for applications when it has a local, computationally stable basis and when this feature is preserved under knot insertion. These requirements are equivalent to saying that the space must have an optimal normalized totally positive basis. Unfortunately, not all generalized spline spaces admit such a basis, and, even when this is the case, its computation may be very challenging. In this talk we present a simple and general method for the construction and computation of the optimal normalized totally positive basis and we show how the proposed approach may be used to establish whether such basis exists in a given spline space.